

✓ **Problem M.1: (10 points)**

A DT system is described through the following difference equation:

$$y[n+2] = \sqrt{2}y[n+1] - y[n] + x[n+2].$$

- (a) Provide in detail all “other” representations of the system above and show your relevant work.
- (b) Is the system causal? Explain.
- (c) Is the system stable? Explain.

✓ **Problem M.2: (10 points)**

Find all signals $x[n]$'s with the following expression of $X(z)$ and indicate for each one whether it is stable and causal.

$$X(z) = \frac{z^4 - 1}{z^3 - 5z^2 + 8z - 4}.$$

✓ **Problem M.3: (15 points)**

You are told that the Z-transform of a function $h[n]$ is given by

$$H(z) = \cos(z).$$

- (a) What are its possible ROC's (Identify its point of singularities)?
- (b) Using contour integration find all possible expressions of $h[n]$.
- (c) Explain and show how you would have obtained the answer through a different method.
- (d) If $H(z)$ were the system function of a DT LTI system, is the system causal? Is it stable? Explain.

✓ **Problem M.4: (10 points)**

Find the Z-transform of

$$x[n] = \delta[n] + \frac{\sin(\omega_o n)}{\omega_o n} u[n-1],$$

where ω_o is some positive integer.

✓ **Problem M.5: (10 points)**

Compute the convolution of the following two signals:

$$x[n] = \cos(\omega_o n) \quad y[n] = a^n u[n],$$

where ω_o is some positive constant.

✓ **Problem M.6: (15 points)**

A DT causal LTI system has a rational system function $H(z)$ that is known to have one simple pole at "1" and no zeros. It is also known that if its input is $x[n] = 2^n$ for all n , then $y[n] = 2^{n+1}$ for all n .

- (a) Determine its impulse response.

Since this system is unstable, you decide to "stabilize" it as follows: You are to feed the output of $H(z)$ as an input to another system (i.e., in series).

- (b) Is it possible to obtain an overall stable system? If yes, explain how and if no, explain why.
- (c) If your answer is "yes", is your solution "implementable" ? Explain.

Alternatively, you decide to "stabilize" $H(z)$ as follows: You are to feed the input of $H(z)$ as an input to another system (i.e., in parallel) and add their outputs.

- (d) Is it possible to obtain an overall stable system? If yes, explain how and if no, explain why.
- (e) If your answer is "yes", is your solution "implementable" ? Explain.